One Way ANOVA

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**Introduction**

One-way ANOVA.

One-way **AN**alysis Of **VA**-iance

- Extension of independent samples t-test to 3 or more samples.
- Hypothesis testing only – Confidence intervals are not provided with this method.

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**Example**

Compare efficacy of treating people with high cholesterol with 20 mg. vs. 40 mg. of Lipitor on reduction of cholesterol.
**Conditions to Reject Null**

1. $\mu_1$ and $\mu_2$ are equal, but $\mu_i$ is different from both of them.
2. $\mu_1$ and $\mu_2$ are equal, but $\mu_i$ is different from both of them.
3. $\mu_1$ and $\mu_2$ are equal, but $\mu_i$ is different from both of them.
4. $\mu_1$, $\mu_2$, and $\mu_i$ are all different from each other.

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**F Statistic**

Nomenclature:
- $MS_b$: Mean Sum of Squares between
  - Sometimes called $MS_e$
- $MS_w$: Mean Sum of Squares within
  - Sometimes called $MS_e$

Obtained $F = \frac{MS_b}{MS_w}$

Mean sum of squares within:

F Table in Appendix C of Textbook

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**Mean Sum of Squares Between**

F-statistic, because it is a ratio, has not one, but two degrees of freedom.

$$MS_b = \frac{SS_b}{k-1}$$

$SS_b = \sum (\text{Sum of Squares between})$

$k$ = Number of Groups being compared
Mean Sum of Squares Within

\[ MS_w = \frac{\left( \sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2 \right)}{n_k} \]

\[ \left( \sum x_i^2 - \left( \frac{\sum x_i}{n} \right)^2 \right) \]

\[ \left( \sum x_i - \left( \frac{\sum x_i}{n} \right) \right) \]

\[ (N-k) \]

Note: \( N = n_1 + n_2 + n_3 \)

What MSw Represents

- The variance of a group of observations does not change when the same constant is added to every observed value.
- \( MS_w \) is an estimate of the variance of the population from which the samples are drawn.
- Whether the null hypothesis is true or false has no effect on \( MS_w \).
What MSb Represents

- MSb is a measure of variability of the means of the groups.
- The equation for the Mean Sum of Squares between: \( MS_b = SS_b / k - 1 \)

Adding a Constant

If we add 10 to all observations in one group, but not another, as we did in the preceding Self-Assessment exercise, what effect would this have on MSb?

MSb would increase AND F > 1

Critical F and F-test

- \( k = 3 \), \( N = 3 \times 4 = 12 \)
- Numerator \( df = k - 1 = 2 \)
- Denominator \( df = N - k = 12 - 3 = 9 \)
- \( \alpha = .05 \)
Example One Way ANOVA

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

One Way ANOVA

Example (cont)

\[ MS_E = \frac{\sum (X^2)}{k} \]

\[ MS_E = 13.25 \]

One Way ANOVA

Example (cont)

\[ MS_E = \frac{\sum (X^2) - \frac{1}{k} (\sum X)^2}{k(k-1)} \]

\[ MS_E = 16.1833 \]

Example (cont)

Obtained \( F = \frac{16.1833}{13.25} = 1.221 \)

Numerator df=k-1=4-1=3

Denominator df=N-k=20-4=16

Critical \( F = 3.24 \)

One Way ANOVA
Unequal Sample Sizes

\[ SS_b = \left( \frac{\sum X_1}{n_1} \right)^2 - \left( \frac{\sum X_2}{n_2} \right)^2 - \ldots - \left( \frac{\sum X_k}{n_k} \right)^2 - \frac{\sum X_1}{N} \]

Two-Way ANOVAS

- Two-way ANOVA – Adds another dimension to categorization of groups.

<table>
<thead>
<tr>
<th>Tall women</th>
<th>Med women</th>
<th>Short women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall men</td>
<td>Med men</td>
<td>Short men</td>
</tr>
</tbody>
</table>

Example: gender and height

Computing Two, Three and Factorial ANOVAS

- Compare the variance between groups and within groups.
- For each factor, calculate similar to one-way ANOVA
- For the interaction effects, the df interaction = df factor ‘a’ x df factor ‘b’

\[ df_a \times df_b \]
Review

- Hypothesis testing compares
  Observed and Critical F's.